GIREP

Conference 2006 - Modelling in Physics and Physics Education

Program and Abstracts

August 20-25, Amsterdam, The Netherlands

AMSTEL INSTITUTE
University of Amsterdam

Proceedings
GIREP Conference 2006

August 20 – 25, Amsterdam, Netherlands

Modelling in Physics and Physics Education

To the grand philosophical question: “*What is man*?”
Aristotle answered:
“*Man is a rational animal*.”
Modeling Theory offers a new answer:
“*Man is a modeling animal*!”
Homo modelus!

see Hestenes, p. 34

Editors:
Ed van den Berg
Ton Ellermeijer
Onne Slooten

Organized by:
Groupe Internationale de Recherche sur l'Enseignement de la Physique (GIREP)
AMSTEL Institute, Faculty of Science, Universiteit van Amsterdam

With the support of:
European Physical Society
City of Amsterdam
University of Amsterdam
Association DBK-na
Center for Microcomputer Applications

People in charge
Ton Ellermeijer
Ed van den Berg
AMSTEL Institute
Universiteit van Amsterdam

International Advisory Committee
Michele D'Anna, Switzerland
Ton Ellermeijer, Netherlands
Manfred Euler, Germany
Helmut Kuehnelt, Austria
Marisa Michelini, Italy
Rosa Maria Sperandeo Minea, Italy
Vivien Talisayon, Philippines
Dean Zollman, USA

Local Organizing Committee
Ton Ellermeijer (Chairman)
Ed van den Berg (Co-chairman)
Jasper Bedaux
Daday van den Berg
Margaret Fuller
Piet Geerke
Hans Hulst
Lodewijk Koopman
Ewa Kedzierska
Leentje Molenaar
Matti Molenaar
Piet Molenaar
Cees Mulder
Johan van de Ridder
Frank Schweickert
Diederik Slob
Peter Uylings
Barbara van Ulzen
Chris van Weert
Olga Zika

How to contact GIREP and to become a member:
Secretary: *Gorazd Planinsic*, University of Ljubljana, Faculty of Mathematics and Physics, Jadranska 19, SI-1000 Ljubljana, Slovenia
(fax: +386 (1) 2517281,
E-mail: gorazd.planinsic@fiz.uni-lj.si
Website: http://www.girep.org/

Gemeente Amsterdam

Preface

This book and CD contain almost all papers presented at the 2006 Amsterdam GIREP Conference:

The review process for papers was as follows: The chairperson of each paper session conducted a first review and recommended whether or not a paper should be published in the CD proceedings on the basis of relevance and quality and whether or not the paper should be considered for the book. A second review was conducted by reviewers after the conference. Papers receiving two positive book recommendations were accepted for the book version. Papers with one positive book recommendation went through a third review by the editors. Poster papers were reviewed by one reviewer.

All papers in the final version were posted on the web and authors were asked to check for major errors which might have slipped in somewhere in the word processing towards the final format.

All book papers have also been included on the CD. The CD also includes the photo collection of the conference.

This book and CD constitute one product with one ISBN number. When referring to any paper, whether in the book or on the CD, the reference should be:
[Author name](2008).[paper title]. : E. van den Berg, A.L. Ellermeijer, O. Slooten (Eds.), Modelling in Physics and Physics Education, [page number]. Amsterdam: AMSTEL Institute, University of Amsterdam.

ISBN 978-90-5776-177-5
Proceedings GIREP conference 2006 (August 20 – 25, Amsterdam, Netherlands): Modelling in Physics and Physics Education
Photographs by: Andrea Denotti
Editors: Ed van den Berg, Ton Ellermeijer, Onne Slooten.
Cover: Diederik Slob
Publisher: University of Amsterdam
Printed by: Alphabase, Alphen aan den Rijn, Netherlands

Table of Contents

Action on Stage: Ways to Unify Classical and Quantum Physics Using the Action Model

Action on Stage: Historical Introduction[7]

Jon Ogborn, Jozef Hanc[8]*, Edwin F. Taylor*[9]
Institute of Education, University of London, UK
Institute of Physics, P.J. Safarik University in Kosice, Slovakia,
Department of Physics, Massachusetts Institute of Technology, USA

Abstract

The action principle is a powerful tool for understanding, applying, and building bridges among fields of physics, from quantum theory through relativity to current research. We dramatize those who devised the action principle and its precursors – Fermat, Huygens, Maupertuis, Euler, Hamilton, Einstein and Feynman – with the authors performing the roles of these great physicists and mathematicians. We accept no responsibility for the accuracy of the words of our characters! This is an effort to introduce fundamental physical principles, not to reconstruct the actual historical development of these principles.

Action on Stage

Animateur:

This symposium is about building bridges between things students would like to learn– relativity, quantum theory, particle physics – and things they have to learn – notably classical mechanics. We are interested in simplicity and unity in physics, as well as with exciting students about physics.

Pierre de Fermat

The idea that links these topics is the concept of *stationary action*.

Now we have a problem. Either you know nothing at all about the physical quantity called *action*, or you learned it in a difficult course of theoretical mechanics. Both will make you hostile to our proposals. Either, “I never studied it, so it can’t be important”, or “Anything I don’t understand *must* be too hard for students”.

[7] All pictures were taken from The Mactutor History of Mathematics archive at the website http://turnbull.mcs.st-and.ac.uk/history in the biography index item.

[8] email: <jozef.hanc@upjs.sk>

[9] email: <eftaylor@mit.edu>, website: <http://www.eftaylor.com>

So – we have to tell you about these ideas, starting from zero. How better than to ask the people who invented them to explain what they were doing? First up is the Frenchman Pierre de Fermat.

Pierre Fermat (1601-1665):

Although I never published a scientific paper, my reputation as one a leading mathematician came from my correspondence with other scientists and from them publishing my ideas and methods in their work. I am known primarily for my work in number theory. I also developed analytic geometry independent of Descartes and worked in many other mathematical fields – completely as an amateur.

I had a terrible fight with Descartes. He thought that light is transmitted instantaneously from point to point "like the cane of a blind man," so I had to express my optical theory in terms of "resistance" of different media through which light passes. You have no such difficulty, and Fermat's principle of least time is the oldest variational principle; one that you still use. The idea is simple: the path that light takes is just the one that takes the least time. Among all possible paths, the minimum total time picks out the unique path between fixed initial and final points. What could be easier?

Animateur:

Monsieur Fermat, there's an obvious objection to your idea. How does the light know in advance which path will be the quickest?

Fermat:

When I was alive I could not answer your question. The objection was not overcome until long after my death, when you came to see that every point on an advancing wave acts as a source of little wavelets. Then between point source and point detector the wavelets add up with coherent phase along *the path of stationary time*. I hope you will tell us how some Dutchman figured it out.

Animateur:

We had hoped that, as a Dutchman, Christiaan Huygens could join us here in Amsterdam, but unfortunately he is away at the Royal Court in France. His big idea was that light is a wave, and that where the wave goes next can be predicted by supposing that each point on the wave front acts as a source of little wavelets. The many wavelets all superpose, adding up in constructive interference to generate the new wave front, but canceling in destructive

Christiaan Huygens

interference everywhere else. Centuries later, Richard Feynman was to adapt the same idea to build a new formulation of quantum mechanics: the “many paths” approach.

Now we jump a hundred years, and our next guest is another Frenchman, Pierre Louis de Maupertuis.

Maupertuis (1698-1759):

Pierre de Maupertuis

I am Pierre-Louis Moreau de Maupertuis. My father, a wealthy pirate, gave me every advantage. I led an expedition to Lapland to measure the length of a degree along the Earth’s meridian, proving that the Earth is hamburger-shaped. Its fame led to my becoming president of the Prussian Academy and a favorite in the court of Louis the fifteenth.

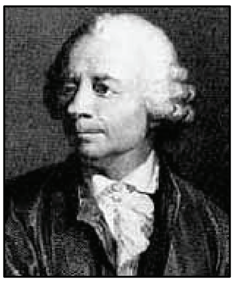
I conceived the principle of least action, that in all events of Nature there is a certain quantity, called *action*, which is always a minimum; that collisions of bodies or refraction of light occur in such a way that the amount of the quantity *mvs* is as little as possible. My original definition of action as the product of mass, speed and distance traveled by a moving object was later restated by my friend Leonhard Euler [see eq. (1)].

My paper was titled, "The laws of motion and rest deduced from the attributes of God" and stated: "Here then is this principle, so wise, so worthy of the Supreme Being: Whenever any change takes place in Nature, the amount of action expended in this change is always the smallest possible." I am horrified to hear that people think that action can sometimes be a saddle point; I reject this idea entirely because the perfection of God is incompatible with anything other than utter simplicity and minimum expenditure of action.

Animateur:

Ignorant people say that nothing of intellectual distinction greater than the cuckoo-clock ever came out of Switzerland. To give them the lie, we now hear from the great Swiss mathematician Leonhard Euler, who supported and developed Maupertuis’ idea.

Leonhard Euler (1707-1783):



Leonhard Euler

It is no boast to say that I am the most prolific mathematician of all time, producing about 900 papers and books in my lifetime. I spent my years largely in the courts of the Tzars of Russia and in the court of Frederick the Great.

Maupertuis is a great buddy of mine, but sloppy in formulating his action principle. I realized that without the law of conservation of energy the action quantity of Maupertuis loses all significance. So I cleaned it up, formulating the principle of least action as an exact dynamical theorem and giving his action a correct mathematical form:

$$W = \int_{\substack{\text{initial}\\ \text{position}}}^{\substack{\text{final}\\ \text{position}}} mv\, \mathrm{d}s \qquad \text{(assume energy conserved),} \qquad (1)$$

(The integral is calculated along a path of a moving particle.)

My statement “since the plan of the universe is the most perfect possible and the work of the wisest possible creator, nothing happens which has not some maximal or minimal property!” was my acknowledgement of Maupertuis as originator of the action principle.

I also developed a simple, intuitive, geometrically understandable way of finding the minimum or stationary action path (see figure 1)

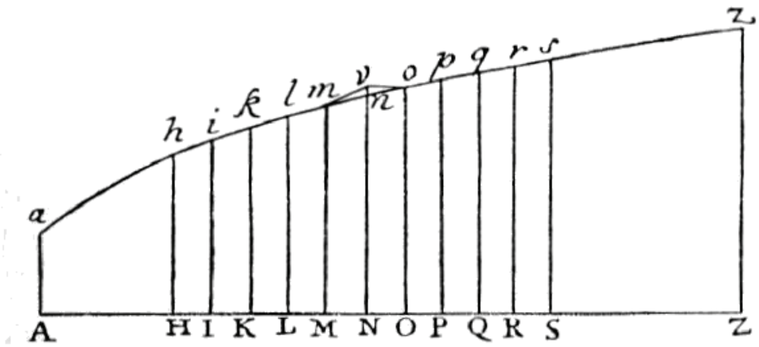


Figure 1: Euler realized that if the action integral is minimal along the entire path, it must also be minimal for every subsection of the path: triplets of nearby points on the path, e.g. mno in my figure. Minimal action means that any change in the path, e.g. point n varied slightly to point v, leads to zero first order change in action. If this condition is be satisfied for each triplet and we go to the limit in which lengths of segments tend to zero, we get a differential equation (the Euler-Lagrange equation), whose solution is the stationary action path.

Later, I helped the career of the young Joseph Louis Lagrange (1736-1813), who wrote to me about his elegant mathematical way to express conditions of minimum action. His ideas led me to drop my intuitive graphical approach and coin the phrase "calculus of variations". Great for mathematics and theoretical physics, but a disaster for physics education! Lagrange's abstract method has dominated your advanced mechanics classes. Too bad, because my graphical method is perfect for modern computers (fig. 2)

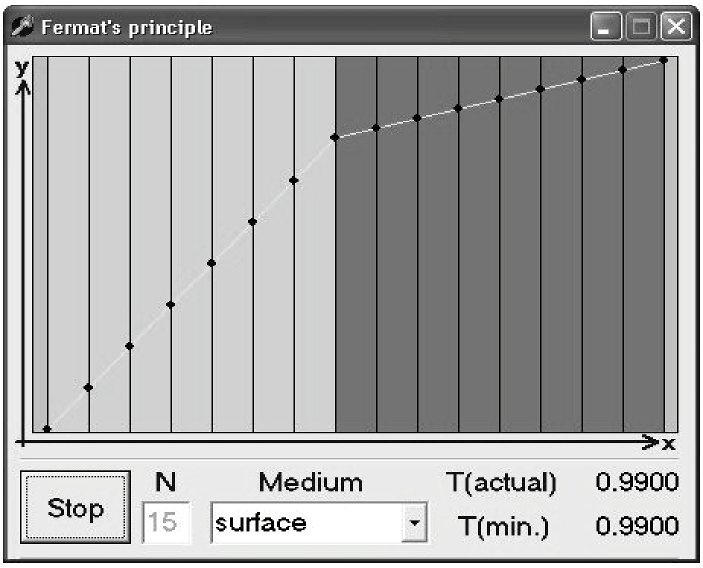


Figure 2: The universality of Euler's graphical approach is demonstrated by this computer display used in modeling Fermat's principle. Click the computer mouse to select an arbitrary moveable intermediate point on the path, then drag the point up and down, looking at the value of the total time, to find the minimal (stationary) time of that point. Then do the same for other points, cycling through them until the time for each results in the least (stationary) value of the total action or time. This method of successive displacements or hunting for the least time path is straightforward but tedious. However the task can be done quickly by computer.

Animateur:

We jump a hundred years again. Here is the Irishman William Rowan Hamilton to tell you about his new version of action, more powerful than ever.

Hamilton (1805-1865):

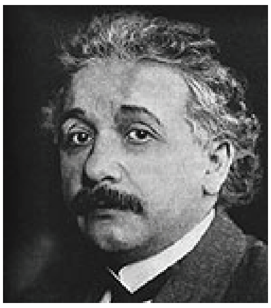
My name is William Rowan Hamilton. I wanted to develop a common mathematical language for particles and waves, so starting from Fermat's least time principle and using the Lagrangian, I found what you call Hamilton's action S:

$$S = \int_{\text{initial event}}^{\text{final event}} L \,\mathrm{d}t = \int_{\text{initial event}}^{\text{final event}} (K - U)\,\mathrm{d}t \qquad (2)$$

William R. Hamilton

Maupertuis' action, remember, determines *trajectories in space* between fixed initial and final locations and requires that energy be conserved. In contrast, my action principle determines *worldliness in space-time* between initial and final events and is true even if the potential energy is a function of time as well as position, in which case the energy of the particle may not be a constant. I understood that action along an actual worldline is not necessarily a minimum but is always stationary compared with action along adjacent alternative worldlines between the same fixed initial and final events. Actually the word “worldline” is a stranger to me; it took Albert Einstein to make the term important. For me it was just the path between fixed places and times – between what Einstein called fixed *events*.

Einstein:



Albert Einstein

I, Albert Einstein, am Swiss by nationality – another blow to cuckoo-clock theory! It was my idea that the physical world has to be structured as space-time events. I emphasized the fact that such events are connected by worldlines in space-time. I could show that the natural, unforced, path from one event to another was that for which ageing – wristwatch time – is a maximum.

In relativity, the Hamilton action S for a free particle is just:

$$S = -mc^2 \int_{\text{initial event}}^{\text{final event}} \mathrm{d}\tau \qquad (3)$$

Minimal (or stationary) action along a real worldline makes the total proper time $\tau = \int d\tau$ maximal (or stationary). Therefore, because of the minus sign in front of the integral, the relativistic principle of least action is the same as the *principle of maximal proper time*, called by Dr. Taylor the *principle of maximal aging*.

Moreover it is not difficult to show that for small velocities Eq. (3) gives the same results as classical nonrelativistic Hamilton action.

Animateur:

You will have noticed that these contributions all came from Europe. But in the last century, American physics blossomed, and one of its finest products was Richard Feynman, who completes our story.

Richard Feynman:

When I was in high school my physics teacher Mr Bader told me that Newton’s laws could be stated not only in the form $F = ma$, but also in the form: “average kinetic energy minus average potential energy is as little as possible for the path of an object going from one point to another.” I next got involved with least action as a PhD student with John Archibald Wheeler.

Richard Feynman

This led to the development of the "many paths" version of quantum mechanics, a third formulation mathematically equivalent to the Schrödinger and Heisenberg versions. It also helped me develop my "*Feynman diagrams*" for doing calculations in quantum electrodynamics. For this work I shared the Nobel Prize with Schwinger and Tomonaga.

So how does a mindless particle recognize the least action path or worldline? Does it smell neighboring paths to find out whether or not they have increased action? According to my formulation, yes! The electron explores all worldlines between source and detector. For each possibility there exists a little rotating stopwatch whose hand, or *arrow*, makes a total number of turns equal to Hamilton's action *S* divided by Planck constant *h* (see Fig. 3). The Lagrangian *L*, divided by *h*, is nothing other than the rate of arrow rotation.

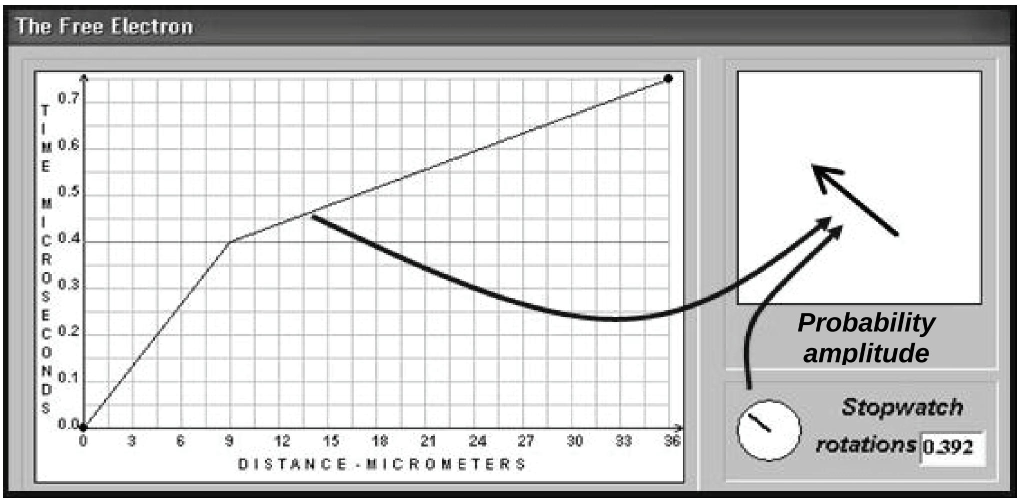


Figure 3: In the "many paths" version of quantum mechanics the electron explores all possible worldlines from initial emission event to final detection event. The figure shows a single one of these worldlines. Along this path a little stopwatch hand rotates at the rate L/h, leading to a contribution to the final amplitude at the detection event.

All these quantum arrows (probability amplitudes) add up constructively (*line up*) if they have similar phases. This is so for worldlines close to the stationary action path (the blue pencil of paths in Fig.4). The arrows cancel out or *curl up* for other sets of worldliness, as you can see in a piece of Dr. Hanc's program. The bigger the mass of an object the narrower is the pencil of nearby worldlines that significantly contributes to the resulting amplitude.

Animateur:

Christiaan Huygens has sent me a letter claiming priority for your "many paths" idea. He claims that it is just his idea of wavelets, in modern clothing. What do you say to that?

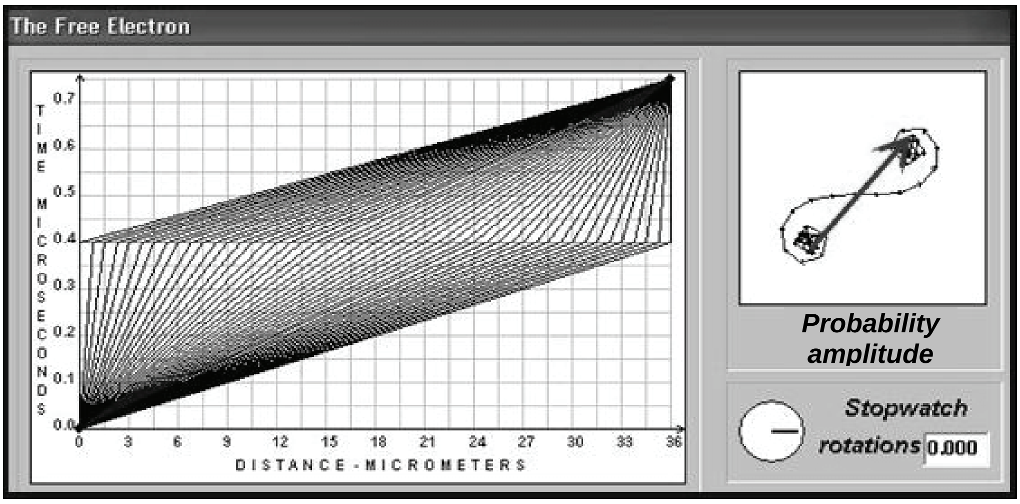


Figure 4: Some of the infinite number of possible worldlines connecting fixed initial and final events. The squared magnitude of the resultant arrow is proportional to the probability of detecting the particle at the final event. As the particle mass increases, the pencil of worldlines contributing significantly to the resultant arrow (shown in blue) becomes narrower and narrower, approaching the single path of classical mechanics.

Richard Feynman:

It's true (I acknowledged Huygens in my PhD thesis). However the use of Planck's constant, and the fact that the idea also works for particles like electrons or atoms, goes beyond Huygens. The idea for the number of quantum stopwatch rotations came from Paul Dirac.

Animateur:

OK, that's it. We will be describing in the papers that follow, how the scalar quantity *action* adds to the physicist's toolkit for analyzing and predicting motion. It looks like this:

1. Use Maupertuis action *W* when we fix in advance the initial and final POSITIONS, and energy is conserved.
2. Use Hamilton action *S* when we fix initial and final EVENTS and energy is may or may not be conserved.
3. Use Newton or Lagrange when we do NOT know where the motion is going from its initial conditions.
4. Use Newton when friction is significant, so vectors are inevitable.

We hope that we have started to break up some of your thought-glaciers about action.

References

Background papers with historical references are available at the web-site:
http://www.eftaylor.com/leastaction.html